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Experimental Researches on Drawn Steel

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PHILOSOPHICAL TRANSACTIONS.

I. *Experimental Researches on Drawn Steel.*

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PART I.

THE INFLUENCE OF CHANGES OF TEMPERATURE ON MAGNETISM.

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1. THE experiments which are included in this part are the outcome of a former investigation, and relate chiefly to the influence of drawing on the magnetism of steel wires and its changes with moderate fluctuations of temperature. The effect of alterations of temperature on the residual magnetism of steel was examined many years ago by WIEDEMANN. His experiments, which have often been repeated, show that on heating a magnet to the temperature of steam much of the magnetism disappears, but that on cooling part of the magnetism so lost is restored; at each repetition of the heating and cooling the permanent loss becomes less and less, and ultimately the magnetic intensity fluctuates between two definite values, higher and lower intensities corresponding to lower and higher temperatures respectively. The

change of intensity in this cyclic state is nearly a linear function of the temperature, and the relation is

$$I_{t'} = I_t \{1 + \alpha (t' - t)\},$$

where $I_{t'}$ and I_t are the magnetic intensities at the higher and lower temperatures t' and t, and α is a coefficient which, in general, is negative.

For a given range of temperature* the irreversible part of the change may be expressed by the equation

$$I_f = I_i (1 + \beta),$$

where I_i and I_f are initial and final intensities. Hitherto, almost without exception, β, for residual magnetism, has been found to be negative, that is to say, there is a permanent loss of magnetism as the result of repetitions of heating and cooling.

The magnitude of both α and β varies considerably, but the conditions which determine the magnitude have not been exhaustively examined. Some of these conditions are investigated here, and it will be shown that under certain well-defined circumstances the coefficients α and β may change sign.

2. In a former paper† it was proved that the dimension ratio of a magnet governing its demagnetising factor controls to a large extent the magnitude and even the sign of the temperature coefficient. The experiments then made were carried out on pianoforte drawn steel in the commercial state, but they have now been extended to steel in other conditions, and the results are given in Table I., from which Diagrams I. and II. are plotted.‡

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* In the experiments in this paper the range of temperature is from 14° to 100° C.

† 'Roy. Soc. Proc.,' vol. 62, p. 210.

‡ Throughout this paper all numerical results are expressed in c.g.s. units and in degrees Centigrade.

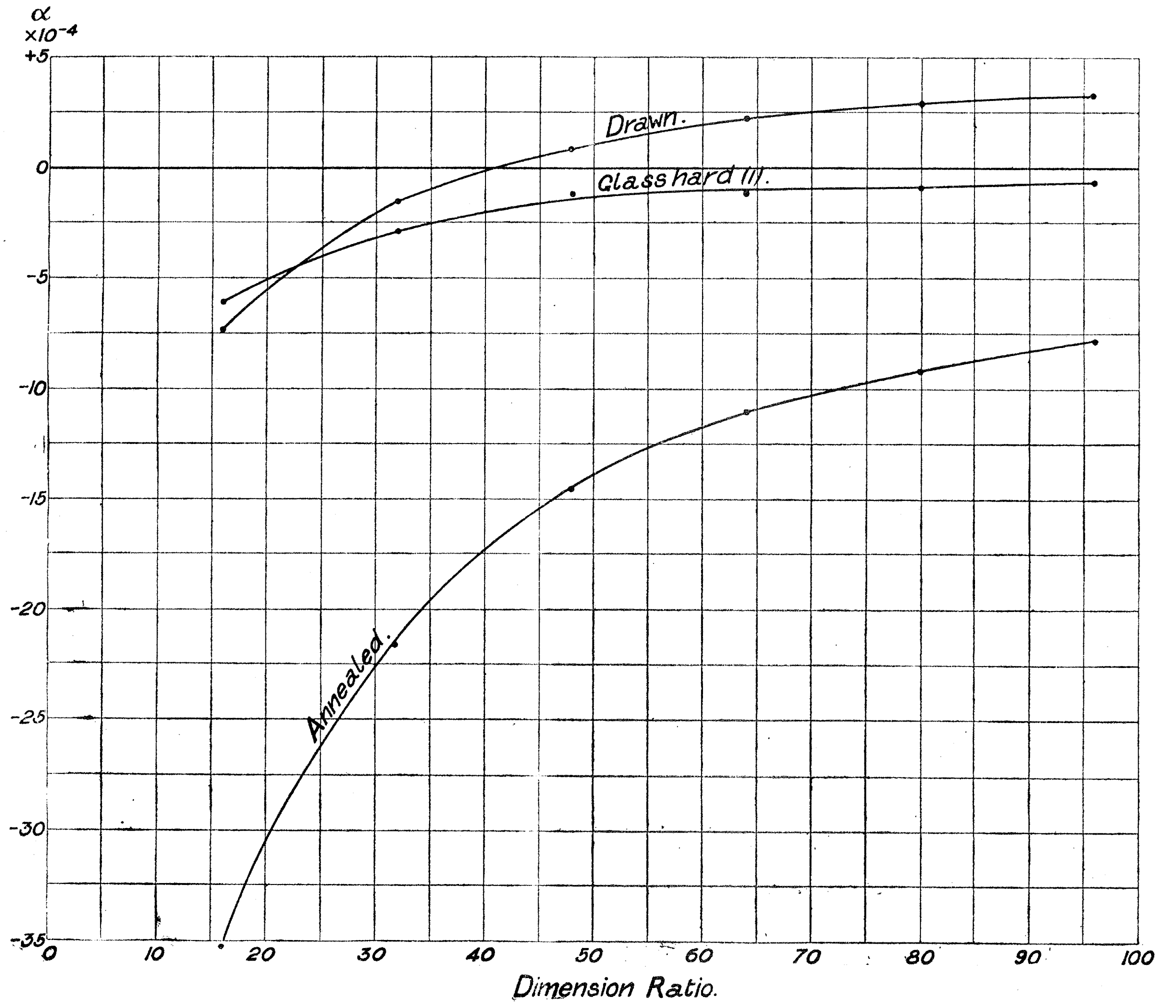
TABLE I.—Relation of Dimension Ratio to Residual Magnetic Intensity and its Temperature Coefficient in Steel Wire.

H 30 Piano Wire.

Length, in centims.	3	6	9	12	15	18
Dimension ratio	16	32	48	64	80	96
Demagnetising factor	0·1120	0·0340	0·0177	0·0106	0·0069	0·0040
Annealed.						
Intensity initially, *i.e.*, before heating and cooling	160	161	276	387	461	525
Intensity finally, *i.e.*, after heating and cooling	36	103	196	290	363	431
Temperature coefficient (α) × 10^{-4}	−35·3	−21·6	−14·5	−11·0	−9·2	−7·8
Permanent loss (β)	− 0·78	− 0·36	− 0·29	− 0·25	−0·22	−0·18
Glass Hard.						
Intensity initially	252	483	551	602	570	580
Intensity finally	216	437	520	562	556	569
Temperature coefficient (α) × 10^{-4}	−6·03	−2·28	−1·17	−1·22	−0·97	−0·55
Permanent loss (β)	−0·15	−0·09	−0·06	−0·07	−0·03	−0·02
Glass Hard, Remagnetised.						
Intensity initially	299	524	608	655	681	659
Intensity finally	286	508	594	643	668	646
Temperature coefficient (α) × 10^{-4}	−5·26	−2·31	−1·47	−1·05	−0·91	−0·82
Permanent loss (β)	−0·04	−0·03	−0·02	−0·02	−0·02	−0·02
Cold Drawn.						
Intensity initially	137	313	483	602	683	727
Intensity finally	79	204	378	514	595	637
Temperature coefficient (α) × 10^{-4}	−7·32	−1·51	+0·84	+2·25	+2·96	+3·17
Permanent loss (β)	−0·40	−0·35	−0·22	−0·15	−0·13	−0·12

Diagram I.

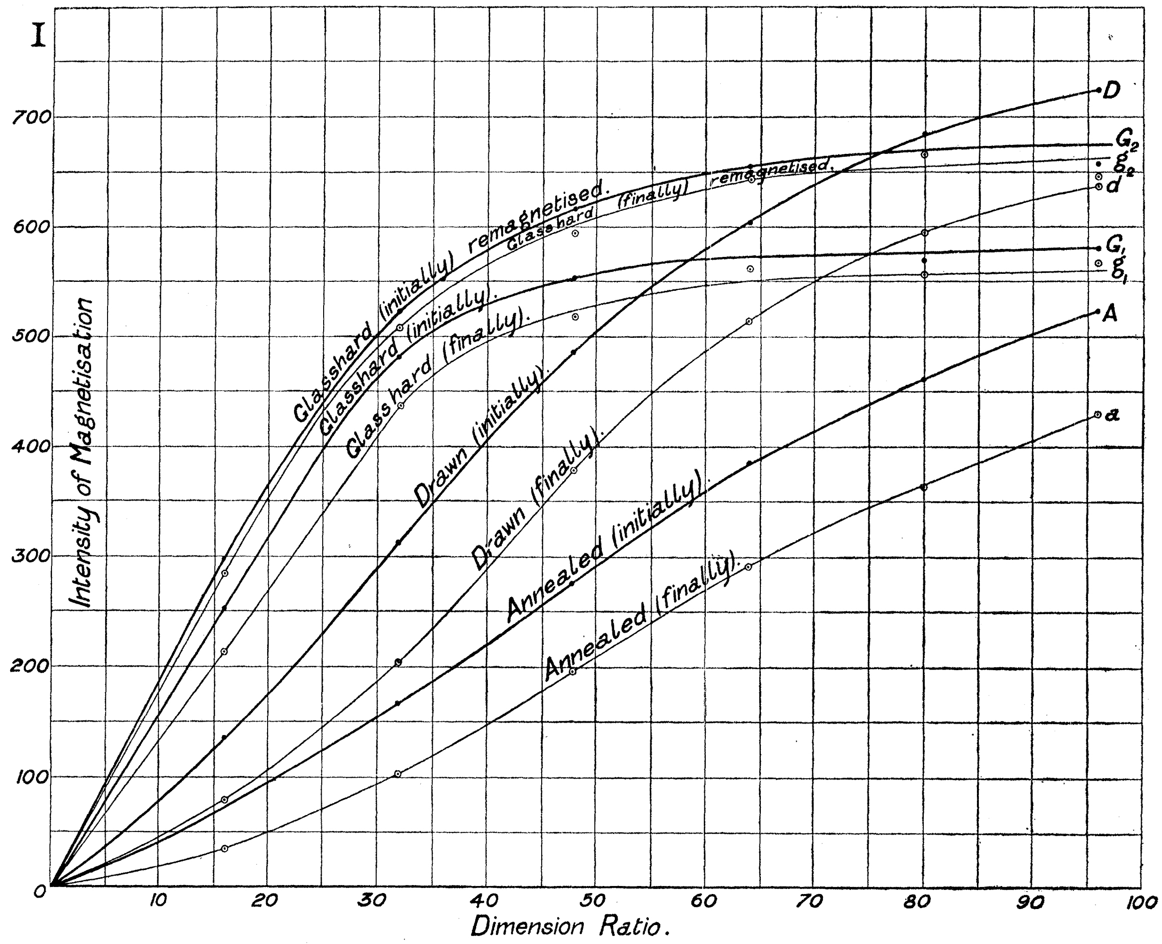
Piano Steel Wire.



The relation of magnetic temperature coefficient (α) to dimension ratio.

Diagram II.

Piano Steel Wire.



The relation of residual magnetic intensity to dimension ratio.

It was also shown, and it can be seen from the table and diagram, that a rise of temperature in the cyclic state increases the magnetic intensity, and a fall of temperature diminishes it in the case of a magnet of large dimension ratio constructed of drawn steel of the kind supplied for pianofortes,* an effect contrary to what is ordinarily observed.

The results of these experiments, and of others which need not here be introduced, upon a number of steels of different chemical composition, as well as the fact that stretching simply does not produce any marked change in the temperature coefficient, strengthen the conjecture that the abnormal effect is due to repeated drawing.

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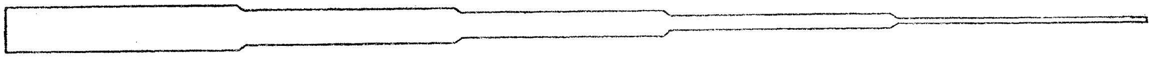
* Knitting needle and pinion wires do not yield this effect, as they are treated differently to music wire in the process of drawing.

3. In order definitely to test this conclusion, it was necessary to procure samples of wires drawn down finer and finer from one original piece, and Messrs. W. SMITH and SONS, of Warrington, kindly undertook to supply them. The first delivery which came to hand was drawn successively from a wire about 0·159 centim. in diameter, the appearance of the wire being like this:—



The material of the whole was one and the same, and the only difference between one part and another was the amount of traction which had been applied. Lengths were cut off from every stage in the drawing, so that each piece was 100 times longer than its diameter, and all were separately magnetised between the poles of a powerful electromagnet and then immediately examined for magnetic properties. The coefficient which, at the first, is incremental (marked in Table II. with the positive sign*), becomes not more so, but less so as the drawing proceeds, and is finally only half as large as at the beginning, and it seems as if extreme traction might ultimately reduce it to zero. This unexpected result indicates that, if drawing produces the abnormal effect, there must be some stage earlier than the first of this series where a maximum incremental coefficient would be developed. It became necessary again to apply to Messrs. W. SMITH and SONS to prepare for me a complete set of wires drawn successively as before, but beginning now at the rough

TABLE II.—Residual Magnetic Intensity and Temperature Coefficient for Successive Amounts of Traction.

Fine Piano Wire.

No.	Diameter.	Dimension ratio.	Residual intensity initially.	Temperature coefficient, α.
	centims.			$\times 10^{-4}$.
7*a*	0·159	100	608	+2·66
8*a*	0·134	100	677	+2·39
9*a*	0·118	100	690	+2·54
10*a*	0·106	100	785	+2·27
11*a*	0·091	100	866	+2·32
12*a*	0·089	100	915	+2·25
13*a*	0·067	100	970	+1·22

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* In a previous paper on this subject ('Roy. Soc. Proc.,' vol. 62, p. 210) an opposite convention was employed in regard to the sign, following an older usage.

rod as received from the rolling mill, and after some delay this set of samples was received. The whole series comprised twelve stages in the manufacture of fine wire as follows:—

4. (1) *The Rolled Rod.*—This is produced from a billet of good Sheffield steel containing less than 1 per cent. of carbon. It is passed whilst hot successively through a number of rolls until its diameter is about 0·5 centim.; the hot rod is finally coiled in a heap, and so cools quickly in the open air.

(2) *Rod Annealed.*—The rod as received from the rolling mill is now annealed by enclosing it in pots from which air is excluded; these pots are heated in a furnace to a bright red heat, at which point the firing is stayed and the fire is allowed to die out. This operation occupies 24 hours.

(3) *Rod Hard Drawn.*—In this stage the annealed rod is forcibly drawn through a perforated plate, which at once reduces the sectional area by about 50 per cent.; the rod now becomes hard.

(4) *Rod Tempered.*—The process to which the rod is next submitted is sometimes called "patenting" or "improving." It is carried out in different ways by different manufacturers, but in these wires it consisted in heating uniformly to a bright red heat in absence of air, and afterwards cooling slowly in a special chamber at a moderate temperature.

(5), (6), (7), (8) *Wire Cold Drawn.*—The tempered wire is drawn through smaller and smaller holes in the draw plate, the sectional area being reduced each time by about 40 per cent. of its preceding value; the diameter in these specimens is in this way diminished to 0·137 centim. at the 8th stage.

(9), (10), (11), (12) The succeeding wires are now all drawn from No. 8 directly and do not pass through every intermediate hole; thus No. 10 is not No. 9 drawn one hole smaller, but is No. 8 reduced at once by a single drawing, and similarly for the others, except, perhaps, No. 12, which probably passed through stage 9 or 10.

These facts relating to the drawing of the last three stages are worthy of notice, as the results of a number of experiments on these wires show some irregularity in the progressive change of their physical properties in the final stages, and the method of drawing in these stages may in part account for the irregularity.

5. The samples illustrating the twelve stages were not long enough to allow the thickest of them to be made more than 50 diameters long, and this fixed the dimension ratio for the whole series, but an additional series, from No. 5 upwards, was cut to a dimension ratio of 100. All the wires were magnetised in the same way between the poles of a powerful electromagnet, and then immediately examined for magnetic intensity, and its changes under variations of temperature, with the apparatus formerly described.* The results, which are given in Table III., and plotted in Diagram III., disclose several interesting facts; thus rolling hot and

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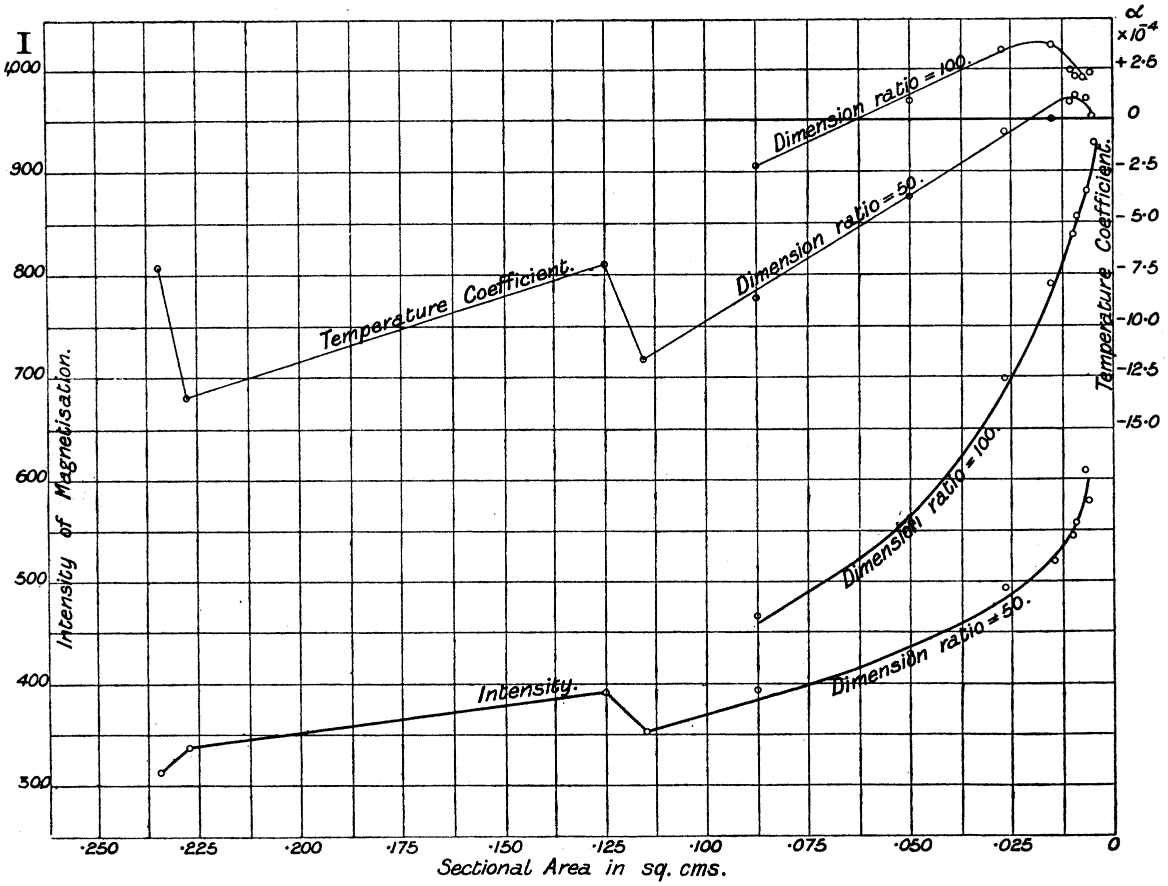
* 'Roy. Soc. Proc.,' vol. 62, p. 210.

TABLE III.—Residual Magnetic Intensity, Temperature Coefficient, and Permanent Loss at Successive Stages in the Drawing of a Steel Rod to Fine Wire.

No.	Condition.	Diameter.	Dimension ratio = 50.			Dimension ratio = 100.		
			Residual intensity initially.	Temperature coefficient, α.	Permanent loss, β.	Residual intensity initially.	Temperature coefficient, α.	Permanent loss, β.
		centims.		$\times 10^{-4}$			$\times 10^{-4}$	
1	Hot rolled .	0·545	315	− 7·11	− 0·110	—	—	—
2	Annealed .	0·539	339	− 12·62	− 0·300	—	—	—
3	Hard drawn	0·399	393	− 6·93	− 0·084	—	—	—
4	Tempered .	0·381	352	− 11·66	− 0·321	—	—	—
5	Cold drawn	0·333	396	− 8·70	− 0·292	469	− 2·35	− 0·076
6	”	0·253	424	− 3·78	− 0·284	551	+ 0·85	− 0·180
7	”	0·185	497	− 0·61	− 0·248	701	+ 3·43	− 0·105
8	”	0·137	519	− 0·19	− 0·207	789	+ 3·82	− 0·032
9	”	0·121	543	+ 0·75	− 0·192	840	+ 2·28	− 0·066
10	”	0·109	557	+ 1·33	− 0·203	857	+ 1·88	− 0·041
11	”	0·099	613	+ 1·25	− 0·155	881	+ 1·99	− 0·016
12	”	0·089	574	± 0·00	− 0·074	930	+ 2·24	− 0·028

Diagram III.

Piano Steel Wire.



The relation of residual magnetic intensity and its temperature coefficient to drawing.

drawing cold both tend to diminish the magnitude of a negative coefficient, whilst drawing the wire several times after tempering, and without re-annealing, completely reverses the sign of the coefficient, which then becomes positive. But extreme drawing bends the curve again to the zero line, and, in the case of the twelfth wire, 50 diameters long, the coefficient actually becomes zero; thus the curve for a dimension ratio of 50 cuts the zero line twice, namely, between the 8th and 9th stages and at the 12th stage.

The diminution of the positive coefficient observed in the experiments on the first set of wires received from Messrs. W. SMITH and SONS is now explained, for it is evident that that series must have commenced beyond the final bend in the curve.

This bend also marks a distinct change in other properties of the wire, for steel wire when drawn too far loses the qualities of strength, elasticity, and electrical conductivity* which moderate drawing confers to a high degree.

The tempering or patenting process in the 4th stage does not seem to be essential to the production, by subsequent drawing, of an incremental coefficient; for if the curve belonging to the larger dimension ratio were continued backwards, following the same path as its companion curve, it nearly, if not quite, reaches the zero line at the 3rd stage, where the wire is drawn *after* annealing, but *before* tempering, and if the experiments had been made on endless wires it is certain that the zero line would have been crossed at the 3rd stage. Hence the production of a positive temperature coefficient is entirely due to cold drawing, if not carried to an extreme stage.

6. In Table III. the initial residual intensities have been calculated as the magnetic moment per unit volume, the volume being obtained from the mass and density. From the figures given in the table, or more clearly from the curve given in Diagram III., it is seen that the intensity steadily mounts upwards as the drawing proceeds. The maximum intensity reached is about 930 units; altogether the residual intensity, after magnetisation, has been increased from 469 at the first drawing to 930 at the last, for magnets 100 diameters long, an increment of no less than 100 per cent. Thus the magnetic properties of steel can be modified to an extraordinary degree by the simple operation of cold drawing through successive holes.

7. Nevertheless, considerable skill and judgment are required in conducting the operation of drawing, if the peculiar qualities which piano wire possesses are to be developed in the highest degree. One fact in connection with the process of manufacture, which may be mentioned here because of its physical interest, is that a wire after drawing through one hole draws more satisfactorily through the next if given a *period of rest* between the operations, and the longer the period of rest, extending even to many weeks, the more satisfactory is the subsequent drawing.†

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* Part II., §§ 2 and 4.

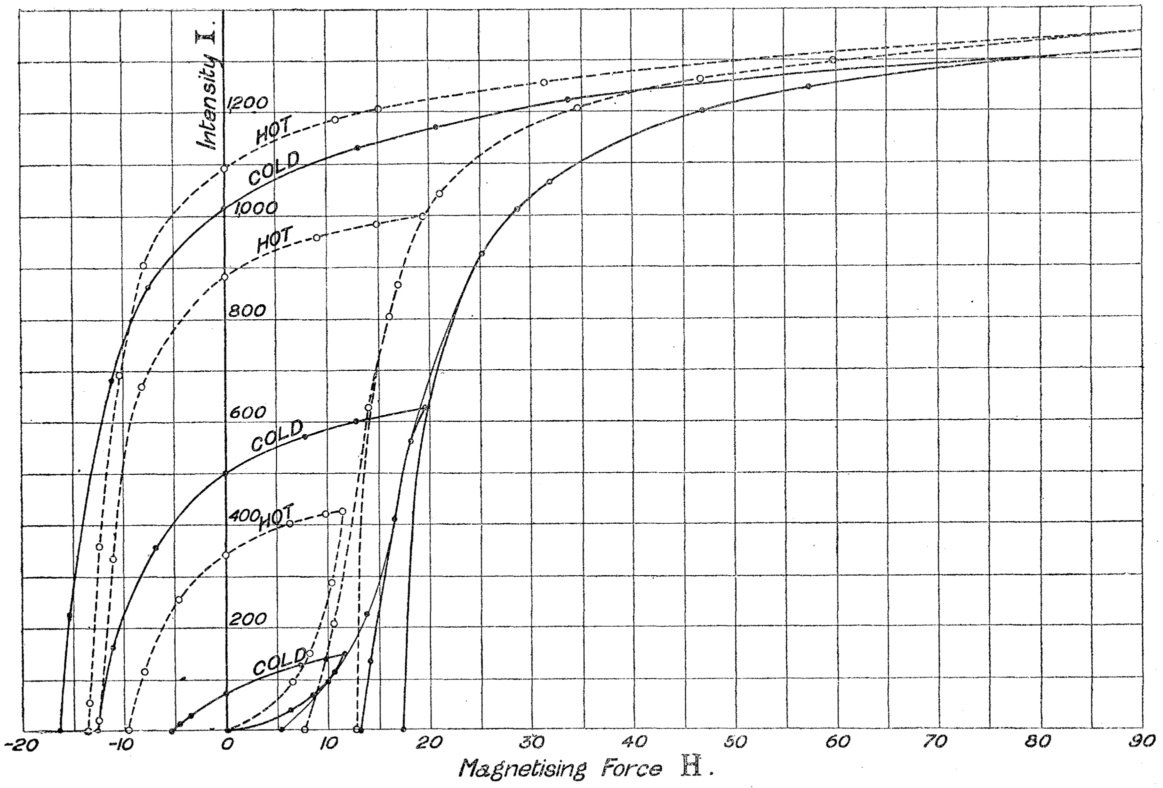
† TOMLINSON remarks that rest after strain diminishes internal friction, 'Roy. Soc. Phil. Trans.,' vol. 177, Part II., p. 835; also vol. 174, p. 5; *vide* EWING, 'Roy. Soc. Proc.,' vol. 30, p. 510.

8. With the object of comparing the magnetisation curves of drawn steel wire when cold and when hot, and also of determining the relation of the temperature coefficient to the intensity, another series of experiments was undertaken. A wire, No. H 30,* was selected of the same diameter and gauge as one upon which a number of experiments had previously been made ;† it was 0·187 centim. in diameter, and its length was 85·3 centims., so that the dimension ratio was 456 and the demagnetising factor negligibly small. The wire was fixed in a glass tube in an upright solenoid, 33·2 centims. from the magnetometer, and the usual arrangements were adopted for tracing the curve of magnetisation according to the one-pole method of EWING. In all the experiments the vertical component of the earth's force was neutralized.

After some preliminary heatings and coolings the wire was carried through a series of graded magnetic cycles at air temperature ; then, after demagnetisation, a current of steam was passed and maintained through the tube, and the same series of graded cycles was repeated, the forces used at either temperature being exactly alike. It will be seen from Diagram IV. that the susceptibility of the hot wire when the force

Diagram IV.

Piano Steel Wire.



Cyclic magnetisations of piano wire at 14° and at 100°

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* Wires marked with the letter H were kindly supplied by Mr. W. D. HOUGHTON, of Warrington.

† 'Roy. Soc. Proc.,' vol. 62, p. 210.

is about 11 or 12 units is nearly three times greater than the susceptibility of the wire cold, and throughout, up to the maximum force employed, the hot curve always lies above the cold curve. On the application of a demagnetising force, after maximum induction, the hot curve droops faster than the cold curve, and crosses it when $-$ H is about 9 or 10 units, and for this force the intensity is the same whether the wire be hot or cold.

9. This suggests that if a suitable demagnetising force be applied the temperature coefficient would be zero, and previous experiments* have verified that a wire of this kind does yield a zero coefficient with an appropriate self-demagnetising force. But it is not possible to calculate with precision, from curves of induction, the requisite demagnetising factor, and consequently the dimension ratio to which such a wire must be cut, in order that the coefficient may be zero, because of complexities in magnetic behaviour arising in part from the irreversible effects of changes of temperature. Nevertheless, an estimate can be made, for, in order that the point of intersection of the curves should lie on the ordinate of no external force, the curves must be sheared by an amount equivalent to about 10 units of force, and as the corresponding intensity is about 800, this would mean that the force per unit of intensity, that is to say the demagnetising factor, must be 0·0125, and in the case of a cylinder the dimension ratio for this factor is nearly 59. Experiment shows, however, that a piece of this kind of wire has an approximately zero coefficient when it is 8 centims. long, and thus the dimension ratio for this condition is 43 instead of 59.

10. Again, the irreversible changes which occur on heating and cooling do not allow the temperature coefficient to be simply calculated from the difference of the hot and cold residual intensities which are left after removal of the magnetising force. Experiments were made on the wire in two ways. Beginning with the wire hot, the intensity fell from 1294 to 1139 during a series of heatings and coolings, and the coefficient was $+$ 0·000358. Then, repeating the experiment, but beginning with the wire cold, the intensity fell from 1204 to 1126, and the coefficient was $+$ 0·000327, a value not very different from the former, but less than half the coefficient calculated from the difference of the hot and cold intensities left immediately after the withdrawal of the magnetising force.

Nevertheless, the relation of these hot and cold curves throws light on the fact that the temperature coefficient of a magnet of drawn steel is positive or negative, according as the demagnetising factor is below or above a certain value, a result which has been fully established (*vide* Table I.).

11. In another series of experiments the temperature coefficient was determined at different stages of the curve, the wire being kept during heating and cooling *under a constant force*. The wire, the same one as before, was, in the first place, demagnetised carefully, and then a field of 11·0 units was applied and maintained; the intensity at 16° was 120·7, but on passing steam through the tube the intensity

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* 'Roy. Soc. Proc.,' vol. 62, p. 210.

immediately rose to 400, more than three times the former amount, and the subsequent alternations of temperature slowly augmented the intensity until a final value was approached at 477; the coefficient then was + 0·000731. Raised another step by a force of 19·1 units, the intensity became 724, and, after heating and cooling a few times, it rose to 993; the coefficient now was + 0·000466. And lastly, with a force of 76·5, the intensity was 1311 initially, and after heating and cooling 1316, the coefficient being + 0·000206. The enormous growth of magnetism under changes of temperature in the earlier stages of magnetisation, and the insignificant increment in the final state, concurrently with the large coefficient when the susceptibility is large and its diminution as the susceptibility becomes less, are the features here principally to notice.

Applying next a small negative force of − 7·19 units, the intensity dropped finally to 745, and the coefficient was then + 0·000229; and it might be anticipated from the position of the point of intersection of the hot and cold curves already examined, that a slightly larger demagnetising force ought to annul the positive coefficient, and that a still larger demagnetising force ought to yield a negative coefficient.

A force of − 11·23 units was next applied and maintained, which, while less than the coercive force for the material hot or cold, was greater than the force at the point where the curves cross. With this force we get the following result:—

Intensity.		
Cold.	Hot.	
+618		Here the south pole was upwards.
	+259	
+227		
	+145	
+134		
	+ 69	
+ 65		
	+ 8	
+ 10		Magnetism reversed.
	− 38	The north pole is now upwards.
− 34		
	− 80	
− 74		
	−116	
−107		
	−151	
−141		

Here it is evident that the force applied has been too large to allow a cyclic state to be established before reversal of the magnetism sets in, yet, in accordance with anticipation, up to the point where the original magnetism is entirely removed the hot value of the intensity intermediate between any two cold values is always less than the mean of the cold values, a result which corresponds to a negative coefficient; later on, after the reversal, the hot is larger than the mean of the adjacent cold values, a result which corresponds to a positive coefficient, and this positive coefficient will, no doubt, continue up to maximum induction.

Both before and after reversal heating appears to produce much larger effects than cooling.

Under alternate applications of heat and cold, the coercive force is greatly lessened, and now lies at less than 11·2, instead of its former value 13·5 hot and 17·0 cold.

12. In the next place the effects of alternate changes of temperature on *residual* magnetism were studied.

The same wire was again employed, and, starting from the demagnetised state, a small force was applied, and the residual magnetism subjected to heating and cooling. Then stronger forces were successively applied and, in the same way, at each step the coefficient was determined; after the maximum had been reached, part of the magnetism was removed, a step at a time, and again the temperature coefficient was examined at each stage. Table IV. is an abstract of the results so obtained.

TABLE IV.—Effects of Heating and Cooling on Residual Magnetism for Progressive Magnetisation.

H 30 Piano Wire.

Magnetising force, H.	Residual intensity.		Temperature coefficient, α.	Permanent change, β.
	Initially, I_i.	Finally, I_f.	$\times 10^{-4}$	$\times 10^{-2}$
10·32	107	91	+ 3·10	− 15·0
19·52	616	572	+ 4·53	− 7·0
23·33	838	794	+ 3·59	− 5·0
(About 100)	1022	967	+ 2·82	− 5·0
(Small negative force)	837	840	+ 4·34	+ 0·4
− 10·54	762	775	+ 4·42	+ 2·0
− 15·03	270	312	+ 3·66	+ 16·0
− 15·93	114	157	+ 1·88	+ 38·0
—	21	64	− 2·90	+ 204·0
—	− 12	+ 30	− 4·70	—
—	− 62	− 20	+ 25·00	− 224·0

From this table the influence of the intensity on the magnitude and sign of the coefficients α and β can be traced.

In the first place, the coefficient rises to an early maximum and then falls to a minimum at the highest intensity; on the return path the coefficient again attains a maximum, which occurs at a higher intensity than before, and after this it continually grows less, and ultimately changes sign and becomes negative. The rise and fall suggest some connexion with susceptibility, for which there is evidence in another group of experiments described later on.

In the second place, when the magnetic intensity is rising, the final intensity, after a series of heatings and coolings, is always less than initially, and β is therefore negative, but on gradually removing successive fractions of the magnetism, then, at each stage, heatings and coolings produce a *gain* of magnetism, and the loss or gain is always much greater the smaller the intensity; indeed, the gain becomes very large at low intensities on the downward path, and at last, when the reversed magnetic field has been increased so far as to leave a small reversed residual magnetic intensity, then heatings and coolings clear this out and restore a small magnetisation of the original kind.

These results could be obtained repeatedly; thus, in the following example, which is a typical one, after magnetising to saturation a reversed force left a residual of 109·2, which was augmented by heating and cooling to 167·2 with a positive coefficient of $+ 3·30 \times 10^{-4}$, a further application of reversed force left a residual of 15·2 in the opposite direction, and then as follows:—

Cold.	Hot.	
− 15·2		
	+ 11·4	Direction of original magnetisation restored.
+ 19·0		
	+ 19·0	
+ 23·0		
	+ 22·2	
+ 24·9		
	+ 24·3	
+ 26·2		The coefficient is negative and equal to − 0·00085.
	+ 24·7	
+ 26·8		

Here there is a change in the direction of the magnetisation due to heating and cooling and a coefficient which is now negative. But the negative coefficient is found to become less negative when the intensity thus restored is greater, as the following table shows:—

Final intensity after reversal.	Coefficient.
+ 23·8	− 0·00120
+ 26·8	− 0·00085
+ 30·4	− 0·00047
+ 63·6	− 0·00029

and no doubt a zero coefficient would be reached for a still higher intensity than 63·6.

If, however, the magnetic force which has been applied is strong enough to leave a residual intensity which, after heatings and coolings, still remains in the reverse direction to the original, the positive coefficient is again established.

13. In the experiments recorded above the intensity was raised a step at a time, after each series of heatings and coolings, and a suspicion might be entertained that the magnetic state at any stage had been seriously disturbed by the heatings and coolings at a preceding stage. The next series of experiments was undertaken to test this question, and accordingly, after any series of heatings and coolings, the wire was completely demagnetised by reversals before the magnetisation was carried a grade higher. Beginning at a low intensity, the magnetisation was thus carried to its highest value by easy steps. In returning, the wire was magnetised strongly and then a small reversed force applied, enough to remove some of the residual magnetism; after heating and cooling, the wire was demagnetised, carried again to its highest intensity, and a larger fraction of the residual magnetism removed, heating and cooling repeated, and so on. In this way the following table was constructed (Table V.).

Table V.—Effects of Heating and Cooling on Residual Magnetism with Demagnetisation between each Step. Relation of Intensity to Temperature Coefficient.

H 30 Piano Wire.

Magnetising force, H.	Induced intensity, I.	Residual intensity.		Temperature coefficient, α.	Permanent change, β.	Susceptibility, κ = I/H.
		Initially, I_i.	Finally, I_f.			
				$\times 10^{-4}$	$\times 10^{-2}$	
8·75	70	25	12	+3·05	−50·8	8·2
12·79	181	112	86	+4·07	−23·1	14·1
15·26	288	247	212	+4·98	−13·9	18·8
18·18	500	476	432	+5·15	− 9·4	27·5
22·44	785	702	653	+5·06	− 7·0	35·0
41·07	1141	915	839	+4·66	− 6·5	27·7
101·20	1334	1028	961	+3·27	− 6·5	13·2
− 8·53	—	810	813	+4·45	+ 0·4	—
− 13·69	—	516	535	+4·37	+ 5·8	—
− 15·03	—	291	331	+4·16	+13·8	—
− 17·05	—	− 17	+ 39	−4·10	—	—

The same general features as before again exhibit themselves, namely, a rise and fall of the temperature coefficient as the intensity proceeds either to a maximum or proceeds to a minimum, the largest value of α occurring earlier for increasing than for decreasing intensities; a loss, β negative, so long as the applied force has been positive and a gain when the force has been negative, the magnitude of the gain or loss being greater the less the intensity; a reversal of the direction of the magnetisation at a low intensity by the operation of changes of temperature; and also a change in the sign of α. But the magnitude of the temperature coefficient and of the irreversible change is decidedly larger in this table than in the former one. Hence, for the production of a magnet of constant intensity constructed of drawn steel wire, it would appear to be advantageous to magnetise step by step, heating and cooling at each step without intermediate demagnetisations up to maximum intensity, and then to remove a small fraction of the magnetism by a reversed force; β is then at its least value.* On the other hand, this gives a larger value for α than if no reversed force had been applied.

14. In order to determine how far these results are due to drawing, it is necessary to have a comparison with similar experiments performed on an iron wire, and this was subsequently done. The iron wire was that which is supplied in commerce as such, but probably it borders on very mild steel; it was carefully annealed, and then submitted to a cycle of magnetisation, at first cold and afterwards hot. The curves of magnetisation intersect in this material when the intensity, for rising forces, is about 800 units; at higher intensities the susceptibility is less hot than cold, and the hot residual lies below the cold residual,† accordingly subsequent heating is found to diminish and cooling to increase the residual magnetism. The coercive force is about 4·0 units cold and a little less when hot.

In the next place the wire, after demagnetisation by reversal, was submitted to a very small force and the force withdrawn, then a series of heatings and coolings was applied, and the permanent loss and coefficient were calculated in the usual way. Again, after demagnetisation, a stronger force was applied and withdrawn and a series of heatings and coolings executed. Repeating these operations a step higher each time and demagnetising between each step, we get the result exhibited in Table VI.

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* *Vide* HOOKHAM, 'Journal Inst. Electr. Engineers,' vol. 18, p. 688.

† BAUR, 'Wied Ann.,' vol. 11, 1880; EWING, 'Phil. Trans.,' vol. 176, Part II., p. 637.

TABLE VI.—Effects of Heating and Cooling on Residual Magnetism with Demagnetisation between each Step. Relation of Intensity to Temperature Coefficient.

Annealed Iron Wire.

Magnetising force, H.	Induced intensity, I.	Residual intensity.		Temperature coefficient, α.	Permanent change, β.	Susceptibility, $\kappa = I/H$.
		Initially, before heating and cooling, I_i.	Finally, after heating and cooling, I_f.			
				$\times 10^{-4}$	$\times 10^{-2}$	
3·03	115	78	75	−2·14	− 3·67	37·9
4·93	346	279	275	−1·49	− 1·57	70·2
6·01	616	527	518	−1·24	− 1·71	102·5
8·53	824	704	690	−1·45	− 2·02	96·6
14·13	1027	855	837	−1·45	− 2·12	72·6
40·70	1248	952	931	−1·40	− 2·14	30·6
87·50	1346	962	941	−1·45	− 2·19	15·4
− 2·69	733	750	752	−1·44	+ 0·35	—
− 3·14	677	699	702	−1·28	+ 0·38	—
− 3·81	218	271	276	−1·68	+ 1·70	—
—	39	100	104	−1·68	+ 4·62	—
− 4·26	− 92	− 24	− 16	+1·63	−35·00	—

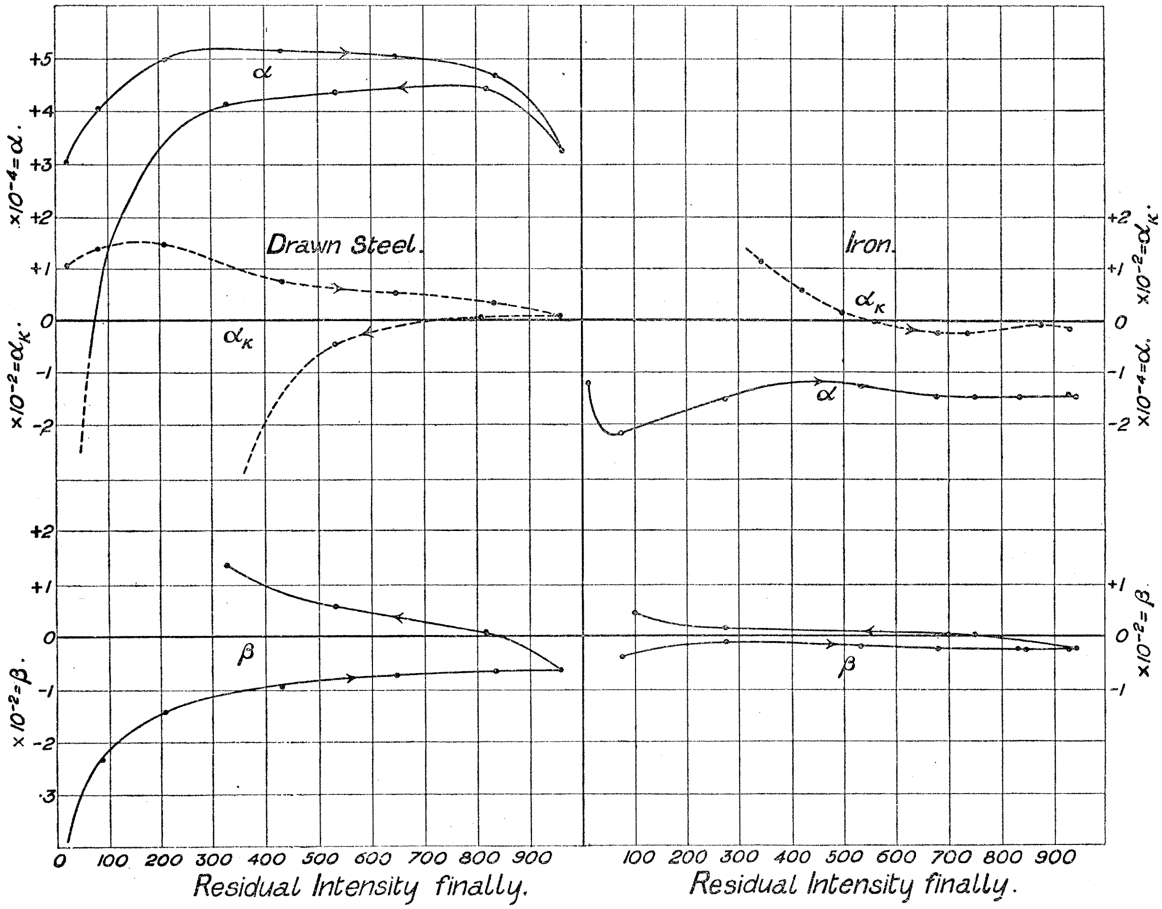
Here the temperature coefficient is negative throughout for rising forces; as the intensity progresses it diminishes to a minimum value when I is about 518 and then, increasing again, becomes nearly constant. For reversed forces the coefficient diminishes to another low value at about 700 units (a higher intensity than before), and then again increases, but if the reversed force is just sufficient to leave a small inverse magnetisation, the coefficient changes sign and becomes *positive.* It appears then that in iron similar features present themselves in the relation of α to I as in drawn steel, but in iron the coefficient is in general negative, and in drawn steel positive, and in each the sign of the coefficient can be reversed when a small inverse magnetisation succeeds a strong direct magnetisation.

The permanent change, β, is larger for low intensities than for high ones, and fluctuates in sympathy with α; for reversed forces it changes sign, that is to say, there is a gain of magnetism due to cyclic temperature changes as there was with drawn sheet. Throughout it will be noticed that in soft iron β, as well as α, is much smaller than in drawn steel. The table shows that a very long soft iron wire may have a temperature coefficient of less than −0·00015 per degree centigrade; also, that if a small reversed force be applied after magnetisation to saturation, no *loss* of residual magnetism takes place due to heating and cooling, but the intensity tends to increase.

15. The experiments which have been narrated, show that the sign and the

magnitude of the temperature coefficient may be inferred from the disposition of the hot and cold curves of magnetisation. This is still further confirmed by reference to Diagram V., where the temperature coefficient of drawn steel, for both ascending

Diagram V.



The relation of temperature coefficients (α) and (α_κ) and permanent change of magnetism (β) to residual intensity.

and descending values of magnetisation, is plotted against the residual intensities left finally after heating and cooling. The broken curve traced underneath is the change of residual magnetism per unit per degree of temperature calculated from curves of residual magnetism when the wire was at 16° and when it was at 100°. This is marked α_κ (as it corresponds to the temperature coefficient of susceptibility for residual magnetism), and is traced for both ascending and descending intensities. The distinctive features of one curve are reproduced in the other, although, as might be expected, owing to irreversible changes, the curves are not an accurate fit. Thus, the zero coefficient experimentally found is displaced largely to the left of the zero position in the calculated curve.

If the iron curves, traced in the same way, are examined, the distinctive features of the one will be found also reproduced in the other, but with a large displacement of one relatively to the other. Thus there is a small value of the negative coefficient in the calculated curve between intensities of 800 and 900, and a maximum about an intensity of 700; the counterpart to these occur in the experimental curve between intensities of 400 and 500, and at less than 100 respectively. Still more interesting is the large displacement of the zero coefficient. In the α_κ curve the zero state occurs about an intensity of 560, and in the α curve probably at an intensity of only a few units. Although I have attempted to obtain experimentally the zero temperature coefficient in iron, I have not succeeded unmistakably, partly because of the general difficulty of working at very low intensities, and partly because of the special difficulty of clearing out all traces of pre-existing magnetisation, which is a very necessary precaution, and of operating in a field of no force. But I have ascertained that at a very feeble intensity the negative coefficient becomes decidedly less negative, and that the curve tends towards zero at some extremely low magnetisation.

EWING, however, has obtained at a very early stage in the magnetisation of iron a positive, and at a higher but still a very low intensity a zero coefficient.* Although his experiment was not performed on residual magnetism alone, as the vertical component of the earth's force was always in operation in such a way as to tend to increase the magnetisation, and at low intensities its effect would be considerable, yet there is little doubt that at some very low residual intensity iron yields a zero coefficient. There is thus a satisfactory correspondence between the results calculated from the relation of the curves of residual intensity when hot and when cold and the results experimentally found for the temperature coefficient of residual magnetism. Every magnet therefore may have a positive, a negative, or a zero coefficient, unless the hot and cold curves happen to be coincident throughout.

The changes which take place in the magnitude of the permanent loss and gain of magnetism due to a series of heatings and coolings, are shown graphically for both drawn steel and iron on the lower part of Diagram V.

16. The experiments I have selected for description throw light, I think, on many of the numerous results which have been published on the effects of cyclic changes of temperature on magnetism,† and also afford some guiding rules for the construction of magnets of high permanence and with small temperature coefficients.

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* 'Roy. Soc. Phil. Trans.,' vol. 176, p. 633.

† References to papers on the "Influence of Changes of Temperature on Magnetism" and allied subjects:—

FARADAY, 'Phil. Mag.,' vol. 8, p. 177, 1836; KATER, 'Roy. Soc. Phil. Trans.,' 1821; BARLOW and BONNYCASTLE, 'Roy. Soc. Phil. Trans.,' 1822; RIESS and MOSER, 'Pogg. Ann.,' vol. 17, p. 425, 1829; KUPFFER, 'Kastner's Archiv,' vol. 6; LAMONT'S 'Magnetismus'; SCORESBY, 'Edin. Phil. Trans.,' vol. 9, p. 254; WIEDEMANN, 'Pogg. Ann.,' vol. 103, p. 563, 1858; MAURITIUS, 'Pogg. Ann.,' 1863, 'Phil. Mag.,' 1864; GORE, 'Phil. Mag.,' 1869 and 1870; GORDON and NEWALL, 'Phil. Mag.,' vol. 42, p. 335, 1871; WHIPPLE, 'Roy. Soc. Proc.,' 1877; ROWLAND, 'Phil. Mag.,' vol. 48, p. 321, 1874; FAVÉ, 'C.R.,' vol. 82, p. 276, 1876; GAUGAIN, 'C.R.,' vol. 80, p. 297, vol. 82, p. 685, vol. 83, p. 896, vol. 85, pp. 219, When the magnet is short relatively to its thickness, the self-demagnetising force will have so preponderating an influence, that it will be advisable, in order to reduce its effect to a minimum, to choose a material of small susceptibility; a hard steel is thus preferred. If, however, the magnet is long and thin, attention must be paid chiefly to the quality and treatment of the material, so that it may develop that condition in which the hot and cold curves of its magnetisation are separated as little as possible.

And further, it has been shown that drawing influences the disposition of the hot and cold curves in such a way that it affords an effective method of regulating the magnitude and sign of the temperature coefficient.

Time Tests

On the Constancy of Magnets with Negligible Coefficients.

17. At the conclusion of a previous paper* reciting experiments upon the construction of magnets with zero temperature coefficients, a brief note was added on the question of the constancy of the zero state with lapse of time. This is obviously important in the application of such magnets to the work of an observatory, and it has therefore received some attention.

In May, 1897, a magnet was constructed of a piece of H 30 wire, 0·187 centim in diameter, and from a previous series of experiments upon this kind of wire it was calculated that it should be cut to a length of 8 centims., having a dimension ratio of 42·6, in order to yield a zero coefficient. It was then magnetised and heated and cooled about twenty times so as to reduce the magnetism to a settled state. The intensity before heating and cooling was 474·4 c.g.s. units, calculating this here, as elsewhere, as the magnetic moment per unit volume and, after heating and cooling, the intensity was 28·1 per cent. less; the coefficient, α, was very small and positive, its value being $+0·000015$ per degree centigrade. This magnet was tested at intervals for the next three years, and its history is given in Table VII., and also in the diagram constructed from this table (Diagram VI.). After the first test it

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615, 1014, and vol. 86, p. 536; Jamin and Gaugain, 'C.R.,' 1876, 'Phil. Mag.,' 1876; Poloni, 'Wied. Beibl.,' 1878; Wassmuth, 'Wien. Ber.,' 1880–02; Baur, 'Wied. Ann.,' vol. 11, 1880; Brown, 'Phil. Mag.,' vol. 23, pp. 293, 420, 1887; Cheesman, 'Wied. Ann.,' vol. 15, p. 204, vol. 16, p. 712; Barus and Strouhal, 'Wied. Ann.,' vol. 20, p. 662, 1883; 'Bulletin U.S. Geol. Survey,' No. 14, 1885; Barus, 'Phil. Mag.,' Nov., 1888; Gray, 'Phil. Mag.,' vol. 6, p. 321, 1878; Bosanquet, 'Phil. Mag.,' vol. 19, p. 57, 1885; Cancani, 'Atti della R. Acc. dei Lincei' (4), vol. 3, p. 501, 1887; Morris, D. K., 'Phil. Mag.,' vol. 44, p. 213, 1897; Durward, A., 'Am. Journal of Sci.,' April, 1898; Pierce, B. O., 'Am. Journal of Sci.,' May, 1898; Ewing, 'Roy. Soc. Phil. Trans.,' vol. 176, 1885; also Canton and Hallström, Coulomb, Hansteen, Christie, Lloyd, Chree, &c.

* 'Roy. Soc. Proc.,' vol. 62, p. 210.

TABLE VII.—Influence of Time on the Temperature Coefficient and Residual Magnetic Intensity of Drawn Steel.

H 30 Piano Wire, 8 centims. long.

Date.	Residual intensity.		Temperature coefficient, α.	Remarks.
	Initially.	Finally.		
			$\times 10^{-4}$	
1897.				
May 21	474	341	+0·15	Magnetised for the first time.
,, 26	—	—	—	Boiled for 2 hours.
June 1	—	—	—	,, ,, 1½ ,,
,, 9	265	264	+0·15	
July 5	255	255	−0·14	
September 10	240	240	−0·40	
October 18	222	—	−0·40	
1898.				
July 11	222	222	−0·47	
,, 11	475	349	+0·05	Remagnetised.
,, 14	341	338	+0·12	Boiled for 4½ hours.
,, 17	—	—	—	,, ,, 12 ,,
,, 18	333	330	±0·00	
October 10	324	320	+0·09	
,, 14	310	307	+0·07	
1899.				
June 16	322	322	+0·61	
October 30	317	316	+0·58	
1900.				
March 29	312	311	+0·42	

Second Sample.

H 30 Piano Wire, 8 centims. long.

Date.	Residual intensity, Initially.	Residual intensity, Finally.	Temperature coefficient, α.	Remarks.
1898.				
October 14	474	—	—	Magnetised first time. Heated and cooled 12 times.
,, 21	368	364	+0·33	Remagnetised. Boiled for 3 hours.
,, 22	—	—	—	Heated and cooled 12 times.
November 9	345	321	+0·31	
,, 14	335	334	+0·32	
1899.				
June 16	341	337	+0·76	
November 3	332	331	+0·65	
1900.				
April 9	331	327	+0·59	

was boiled for three and a half hours in two stages, and a week later the coefficient was exactly the same as before. It was now laid aside for a month, when the coefficient was found to have changed from +0·000015 to −0·000014, and this again slowly altered for twelve months, and became finally nearly steady at − 0·000047, which is less than a half of 1 per cent. for 100° C. The intensity had also changed, at first quickly, but latterly very slowly. Here it is interesting to notice that at any stage heating and cooling through a range of 80° or 90° diminishes the intensity in the later tests to a very trifling extent, but the undisturbed action of time produces a very slow but steady diminution to the final limit; and this recalls the circumstance mentioned previously, how the action of time alone alters the molecular structure of steel so that its drawing qualities are greatly improved (§ 7).

The magnet was now remagnetised (July 11th, 1898), which immediately raised the intensity from 221·7 to 474·8, a value practically identical with its initial intensity; after a series of heatings and coolings, with boiling at intervals, the magnet was laid aside, its coefficient then (October 14th) being + 0·000007, and the intensity 306·9. At the same time another piece of H 30 wire was cut so as to be 8 centims. long, magnetised and tested. Its intensity initially was 474·7 and its coefficient +0·000033. After a number of heatings and coolings, remagnetisation and boiling for several hours, it was tested again, and laid aside for comparison with the former. Its coefficient was then + 0·000032, and the intensity 333·9 (Table VII.). Both these magnets were tested in June and November, 1899, and again in April, 1900. It appears that in the summer of 1899 the coefficients were larger than seven months previously, and reached a maximum of + 0·000061 and + 0·000076, and that since then they have become a little less, and now (April, 1900) stand at + 0·000042 and + 0·000059.

The intensity after heating and cooling has only fluctuated to the extent of 3 per cent.

Both these magnets are almost exactly alike in all the details of their behaviour, and it will be noticed that the second magnet when magnetised and immediately remagnetised, without any long interval between the magnetisations as with number one, arrives at once at the same condition as the first.

18. Magnets made from this wire with a larger dimension ratio would have higher intensities and yield still more constant results, but then the coefficients would depart considerably from zero, because of the too small demagnetising factor. If, however, a longer piece of the wire be taken, a part of its abnormal properties can be removed by heating to a suitable temperature and quenching, as already shown,* and thus a zero coefficient can be obtained with a smaller demagnetising factor.

Two lengths of 12 centims. each were cut from the same kind and thickness of wire as before, and were heated until just red-hot and quenched in water; they were then magnetised and repeatedly heated and cooled. The coefficient was negative and in magnitude −0·000044, and the intensity about 670 for both, the original intensity

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* 'Roy. Soc. Proc.,' vol. 62, p. 215.

immediately after magnetisation and before heating and cooling being about 700. They were then remagnetised, boiled for some hours, heated and cooled many times, and the coefficient determined; after this they were laid aside for several months. This was in November, 1898. When re-examined in June and November of 1899, and again in March, 1900, very little change had taken place in either the coefficients or the intensities, which were about − 0·000025 and 700 respectively. Both magnets were almost exactly the same in their behaviour in every respect, as will be seen from Table VIII., which gives their history.

TABLE VIII.—Influence of Time on the Temperature Coefficient and Residual Magnetic Intensity of Drawn Steel when Tempered.

H 30 Piano Wire, 12 centims. long, heated to redness and quenched.

Date.	Residual intensity.		Temperature coefficient, α.	Remarks.
	Initially.	Finally.		
			$\times 10^{-4}$	
1898.				Magnetised first time and heated and cooled 12 times.
October 14. . . .	707	682	—	
„ 24. . . .	686	675	$-0 \cdot 44$	
November 9 . . .	740	716	$-0 \cdot 32$	Remagnetised, boiled for $2\frac{1}{2}$ hours, and heated and cooled 12 times.
1899.				
June 16	736	730	$-0 \cdot 26$	
November 8 . . .	718	711	$-0 \cdot 24$	
1900.				
March 29	704	700	$-0 \cdot 25$	

Second Sample.

H 30 Piano Wire, 12 centims. long, heated to redness and quenched.

1898.				Magnetised first time and heated and cooled 12 times.
October 14 . . .	700	680	—	
November 3 . . .	677	669	$-0 \cdot 45$	
„ 9 . . .	742	728	$-0 \cdot 27$	Remagnetised, boiled $2\frac{1}{2}$ hours, heated and cooled 12 times.
1899.				
June 16.	749	742	$-0 \cdot 32$	
November 8 . . .	726	721	$-0 \cdot 21$	
„ 15 . . .	—	—	—	Boiled 3 hours.
„ 15 . . .	724	720	$-0 \cdot 25$	
1900.				
April 9	709	709	$-0 \cdot 24$	

For steady values of α and I this latter method of constructing magnets is to be preferred, and with further experimental experience even the small coefficient here exhibited might yet be reduced; the high intensity, too, which is yielded by this method of producing approximately zero coefficients is another advantage. The magnetic moment of these magnets was about 227 each for a weight of 2·59 grammes.

It will be noticed that, in the magnets which have been constructed to have nearly zero coefficients by cutting them to a suitable dimension ratio, there is a tendency for the positive coefficient to grow less and become negative as the intensity declines, and *vice versâ*. This is in accord with the results already found for the relation of the hot and cold curves, where larger and smaller values of the intensities than those corresponding to the intersection of the curves give positive and negative coefficients respectively.

Indeed, in *any* magnet we should expect a change of intensity to produce a change of the coefficient if the latter is dependent on the disposition of the hot and cold curves of magnetisation, and hence the decay of magnetism with the lapse of time, or the increment of magnetism which takes place on remagnetisation, will tend to alter the magnitude of the temperature coefficient. In the latter case such effects have been noticed.*

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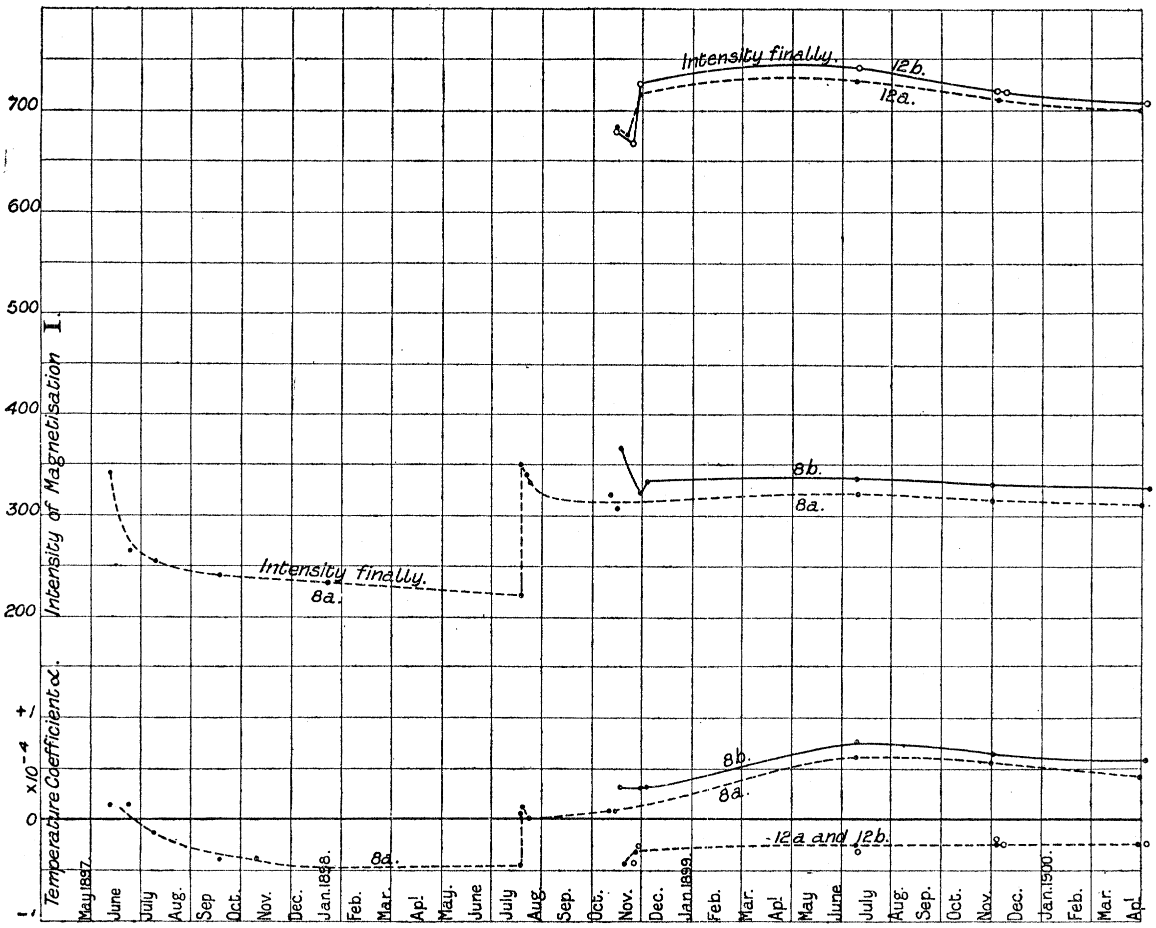
* *Vide* CHREE, 'Roy. Soc. Proc.,' vol. 65, p. 375.

Diagram VI.



Change of magnetic intensity and temperature coefficient with lapse of time in drawn and tempered piano steel.

Nos. 8*a* and 8*b* refer to the first and second samples of piano wire in the commercial state, 8 centims. long.
Nos. 12*a* and 12*b* refer to the first and second samples of tempered piano wire, 12 centims. long.

PART II.

Resistivity, Elasticity, and Density, and the Temperature Coefficients of Resistivity and Elasticity.

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1. Introductory 26
2. The relation of resistivity (ρ) to drawing 26
3. The relation of the temperature coefficient of resistivity (α) to drawing 28
4. The relation of Young's modulus (Y) to drawing 29
5. The relation of the temperature coefficient of Young's modulus (γ) to drawing . . . 31
6. The relation of density to drawing and the influence of density on Young's modulus and on magnetic intensity 33

1. The magnetic behaviour of repeatedly drawn steel wire led to the suggestion that some of the other physical properties of such wire would also exhibit interesting changes. Besides, it was worth while to attempt to trace broadly some connection between one property and another, since, whilst no chemical change presumably takes place by drawing, yet the physical properties might be considerably modified.

The selfsame wires which were employed in determining the change of magnetic properties, and which have been numbered (1) to (12), have been used in the experiments about to be described on resistivity, elasticity, and density, in order to remove any doubt which might be entertained that the material of any one of these specimens was not identically the same during the different tests for its several properties. This unquestionably added to the experimental difficulties, for a length or thickness suitable under one set of conditions was not so suitable under other conditions. Thus, in the determination of Young's modulus, the method of flexure had to be employed which, under other circumstances, would not have been adopted.

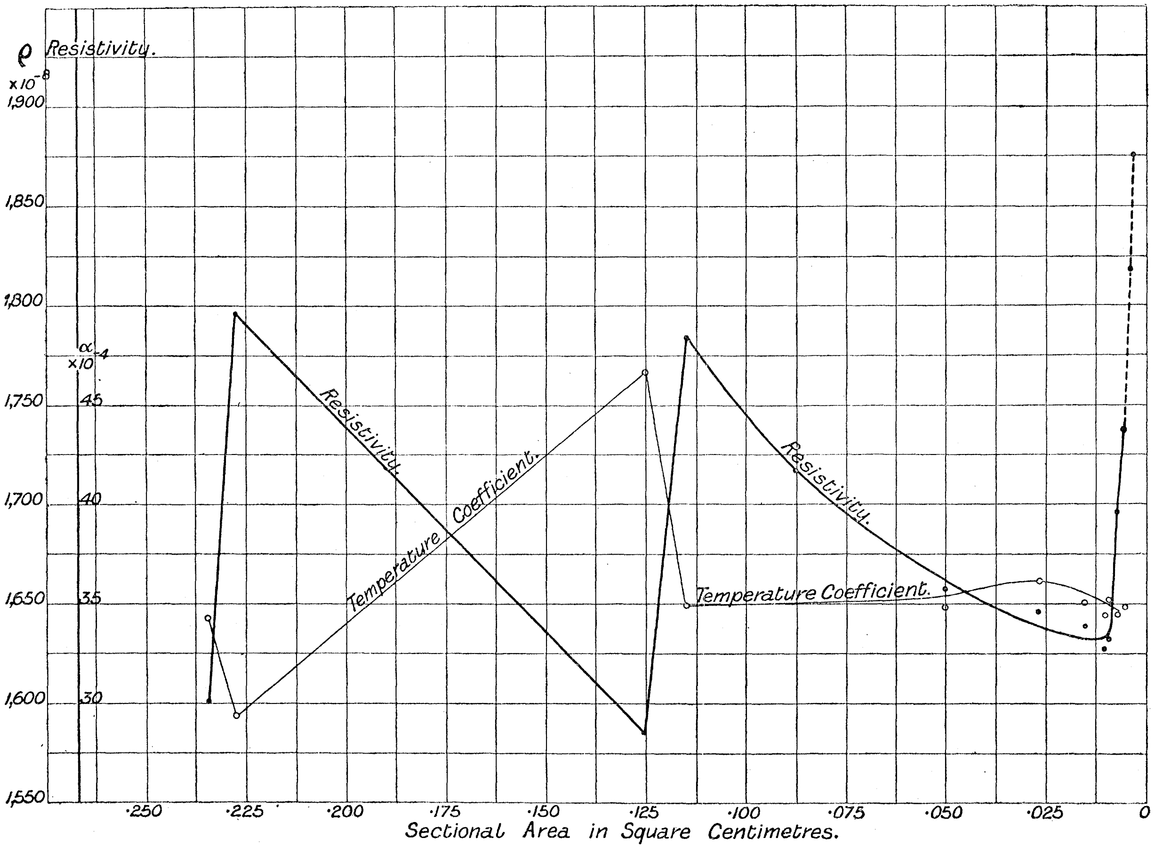
As the methods used here for the determination of resistivity, elasticity, and density are those commonly employed, it will be unnecessary to describe at length the course of the experiments, and the present part will be confined to a brief recital of results.

Resistivity.

2. The resistivity of the wires was observed by comparing the difference of potential at the ends of a known resistance with the difference of potential between two points along the wire when the same current was flowing through the wire and the known resistance. Each wire has been tested by two, and in some cases by three, independent experiments, in which as much variation was introduced as the

apparatus would allow, and the results in the table subjoined are the mean values of the experiments, which, however, differed very little among themselves. The principal source of error lay in the measurement of the diameters of the wires, for in one or two cases the wires appeared to be conical in shape, and in two cases were slightly elliptical in cross-section. By measuring the diameter at many places along the length of the wires, and confirming the results by calculation of the diameter from determinations of the density, the average sectional area has been arrived at. The numerical results are given in the third column of Table IX.,* and in Diagram VII.; they are plotted as a curve, the sectional areas of the wires, which may be taken as a scale of traction, being treated as abscissæ. There is a small reduction of diameter on annealing and tempering which is due not to traction, but presumably to the production of a little oxidation, which would be rubbed off afterwards when the wire was cleaned.

Diagram VII.



Relation of resistivity (ρ) and its temperature coefficient (α) to drawing in piano steel wires.

The effects of annealing or tempering upon the resistivity come out in the curve

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* The diameters of the wires are given in Tables II. and III., and are not repeated in Table IX.

as two prominent peaks; either of these processes increases the resistivity by about 12 per cent. upon the initial state. Hard drawing after annealing between stages (2) and (3) brings the wire down to nearly the original condition, and again, after tempering, cold drawing through two or three holes decidedly reduces resistivity, but an unmistakable increase sets in at the 11th and 12th stages. From the last point the curve has been extended by a broken line to include two more points belonging to still finer wires, Nos. 13*a* and 14*a*, which are not, however, of identically the same material as the others. Nevertheless, the broken line emphasizes the fact that the effect of extreme drawing is prejudicial to conductivity. These contrary effects of drawing, it will be remembered, were also in evidence in the curve of magnetic temperature coefficient, and in both curves the change occurs near to the 8th and 9th stages. A length of the steel wire upon which the experiments just described were carried out was subsequently made glass hard, and the resistivity in that state was 2760×10^{-8}, or about 70 per cent. higher than the wire in its state of least resistance. It is worth notice that minimum resistivity occurs in the *hard drawn* and *hot rolled conditions*, and hence the order of resistivity in an ascending scale is: hard drawn or hot rolled; annealed or tempered; glass hard.

Temperature Coefficient of Resistivity.

3. A simple modification of the apparatus described in the last section allowed the temperature coefficient of resistivity to be obtained.

The same wires were used as before. Each was fixed in a trough surrounded by a water-bath, which could be raised in temperature by the application of gas jets from about 16° to 90° C. Readings were made at intervals during the process of heating and cooling, and the usual precautions were taken for the elimination of the effects of thermo-currents.

Two independent sets of observations were taken for nearly all the specimens, and the mean results, which are given in the fourth column of Table IX., have been plotted on the same diagram as the curve of resistivity, so that the two curves may be conveniently compared.

It will be noticed that the smallest value of the temperature coefficient, namely, 0·00294, occurs when the steel is annealed, and the highest value, 0·00466, when hard drawn, and these least and greatest values coincide respectively with the largest and smallest values of the resistivity. This confirms and extends a law which BARUS has shown to be true for the iron carburets, according to which the temperature coefficient of resistivity is approximately inversely as the resistivity;* and in the

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* The relation given by BARUS is $\rho(m+\alpha)=n$, where ρ is the resistivity, α the temperature coefficient, and m and n are constants. 'Bulletin U.S. Geol. Survey,' No. 14, 1885.

curves here plotted a similar relation between temperature coefficient and resistivity in general holds good, the two curves moving oppositely to each other. The change, however, in the magnitude of the temperature coefficient of the repeatedly drawn steel is small compared to the change in resistivity.

The temperature coefficient of this steel made glass hard is only 0·00177, about half the average value in the drawn state; this again is an example of BARUS' law, for the resistivity when glass hard is, as stated, 70 per cent. greater than when hard drawn.

The ascending order of the temperature coefficient is thus: glass hard; annealed; hard drawn; the inverse of resistivity.

YOUNG'S *Modulus.*

4. The elastic properties of steel are known to undergo a considerable change with drawing, and it seemed desirable to discover if longitudinal elasticity was correlated in any way with the electric and magnetic properties of these steel wires, and how it was modified by annealing, tempering, and traction. The determination of YOUNG'S modulus was effected by measuring the amount of flexure of the wires when loaded at the middle, and with the ends resting on rigid supports. The depression produced by loading was observed through a telescope supplied with a micrometer eye-piece, which allowed an exceedingly small interval to be measured with accuracy. After a reading had been taken of the fiducial mark, with no load hanging from the rod except the pan itself, weights were applied one by one, great care being exercised so that there should be a minimum of vibration; the weights were then removed, one at a time, the depression corresponding to these weights at each stage being observed through the telescope. Two, and in some cases three, independent sets of experiments of this kind were performed on each wire, and nearly always with different distances between the supports, and with different increments and amounts of load, and the mean of the several experiments was taken as the final result.

In the formula for the calculation of the modulus the fourth power of the radius appears as one of the factors, and hence an accurate value of the radius is required if errors are to be avoided. As already mentioned,* a considerable number of careful measurements of the diameters of the wires were made, and these were confirmed from determinations of the density.† The values there used have been adopted here.

The fifth column of Table IX. gives YOUNG'S modulus for the twelve specimens of steel upon which the magnetic and electric experiments already described had been carried out. Diagram VIII. exhibits this column of figures as a curve with sectional

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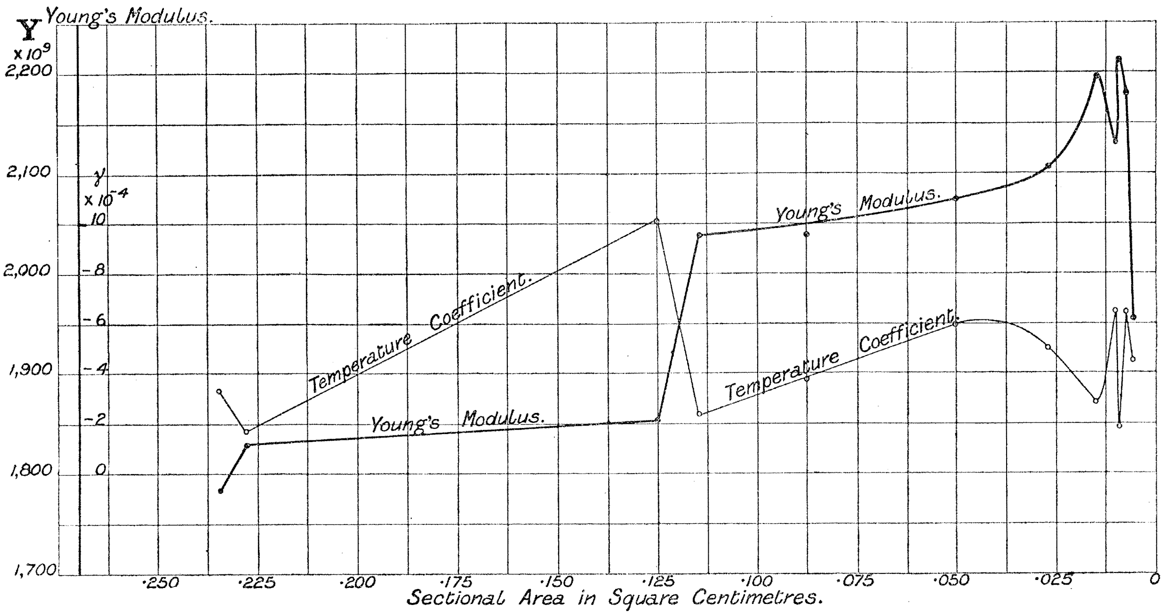
* *Vide* § 2.

† As the deviation of any single determination of the density from its mean never exceeded $\frac{1}{4}$ per cent. in any one specimen, the maximum error on this account in the fourth power of the radius will not be more than double of this.

Diagram VIII.



Relation of Young's modulus (Y) and its temperature coefficient (γ) to drawing in piano steel wires.

areas as abscissæ and Young's modulus as ordinates; on the same diagram the curve of the temperature coefficient of the modulus is plotted, but this will be referred to afterwards.

A feature of this curve is that annealing, hard drawing, and tempering all produce an upward effect, which is continued until the last stages are reached, and then a decided drop occurs; thus, again, the initial effects of drawing are reversed by extreme traction. The increase of the modulus is very conspicuous when the wire has been tempered, the rise at this stage being nearly one half of the whole change, which, from least to greatest, is about 21 per cent. On the other hand, drawing between the 2nd and 3rd stages has only a small effect on the modulus, and the influence of successive cold drawing after tempering for at least two stages is comparatively unimportant; then, after a sharp rise at the 8th and 10th stages, with some irregularity at the 9th, there is the rapid fall to the 11th and 12th points. The irregularity in the final stages is here apparent, as in some of the other curves, and confirms the suspicion that there has probably been some departure from the even course of drawing after the 8th stage, as mentioned when the process of manufacture was described.* It seems not unlikely that it requires very skilful manipulation of the material in order to obtain maximum elasticity, and about stages (8) to (10) the wire possibly develops a critical condition.

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* Part I., § 4.

Temperature Coefficient of YOUNG'S *Modulus.*

5. In determining the temperature coefficient of YOUNG'S modulus the same apparatus as before was used, with the addition of a brass tube in which slots were cut, so that the upper ends of the supports could project into the interior of it. This tube enclosed the steel wire, and another slot of smallest allowable size was cut at the centre of the tube to permit the passage of the suspension for the pan and weights. The slots near the ends were made steam-tight, but, necessarily, this could not be done for the central one, as the suspension wire which passed through had to hang freely. One end of the tube was in connection with a boiler, and this produced a supply of steam which could pass freely along the tube and escape at the other end. No special arrangement, however, was made for the cooling of the tube, the steam was simply shut off and the tube and its contents allowed to grow cold gradually. A thermometer projected into the tube, with the bulb nearly at the centre, and the temperature of the steel was taken to be the reading of the thermometer.

The apparatus thus set up was at first intended for rapid tests, to ascertain whether the *sign* of the coefficient changed at any stage, and less attention was paid to its magnitude, but later on it became possible to obtain numerical results which are worth recording. In some earlier experiments the method was tried of observing the position of the fiducial mark at air temperature, then at steam temperature, and again at air temperature, from which data the coefficient could easily have been deduced. But it was found much better to follow the plan of loading and unloading as already described for determining the modulus, carrying out the observations firstly at air temperature, secondly at steam temperature, again at air temperature, and so on alternately, and calculating the coefficient, γ, from the observed depressions, D_0 and D_t, thus:

$$\gamma = (D_t - D_0)/D_0 \, . \, t.$$

By adopting this method, corrections for expansion of supports, etc., were eliminated. The formula assumes that the change in YOUNG'S modulus is linear and that there are no hysteresis-like effects, but these assumptions are, no doubt, not quite justifiable, although probably not far from the truth.

With rise of temperature the modulus was found to decrease, but on cooling it it was very seldom that it returned exactly to its original value, although, after a repetition of the experiments, the modulus was found to change from one to another of two nearly constant values; thus there is a permanent change before a cyclic state is established. Here is an example:—

Drawn Steel Wire No. 7.

Temperature.	Depression on arbitrary scale (load increment = 100 grammes).	
Cold	145·8	—
Hot	—	147·6
Cold	144·0	—
Hot	—	146·6
Cold	144·0	—

Each of these numbers has been arrived at after a series of loadings and unloadings as described above. Since the modulus is inversely proportional to the depression, the diminution of the latter from 145·8 to 144 means a "permanent" increase of 1·25 per cent. in elasticity.*

The amount of this increase of the modulus varies from stage to stage and roughly follows the variation of the temperature coefficient, large and small values of each being associated. Thus there is a large "permanent" increase when the cyclic change is large. The average amount of the "permanent" increase of the modulus in these wires is about 2·5 per cent. for 80° change of temperature.†

The mean results of the several determinations of the temperature coefficient of YOUNG's modulus on each wire are given in the sixth column of Table IX.

The coefficient is throughout negative, implying that, in the cyclic state, YOUNG's modulus decreases with rise of temperature; the magnitude varies considerably, namely, from $-1·64 \times 10^{-4}$ when annealed, at the 2nd stage, to a maximum of $-10·25 \times 10^{-4}$ at the 3rd stage, when hard drawn. There is a small value again of the coefficient at the 4th stage on tempering, and, after that, the figures for the cold drawn specimens present apparently much irregularity, but if this column of figures be plotted, as in Diagram VIII., we get a curve which repeats in an inverse sense all the features of the modulus curve, so that a relation clearly exists between the modulus and its temperature coefficient. Although the figures in the table do not exhibit a simple inverse proportion between the magnitude of the coefficient and YOUNG's modulus, yet the general statement may be made that larger and smaller values of the modulus are progressively associated respectively with smaller and larger values of its temperature coefficient. This law recalls the similar law connecting the resistivity of the iron carburets and their temperature coefficients, with this difference, that the coefficient for the modulus is negative, whereas the coefficient for resistivity is positive. The average value of the coefficient for YOUNG's

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* This "permanent" effect does not persist indefinitely, but probably disappears in a few days or hours.

† These curious effects of temperature on YOUNG's modulus are in accordance with results published by Mr. SHAKSPEAR in a paper which appeared just after these experiments were carried out. 'Phil. Mag.,' vol. 47, p. 539; also *vide* TOMLINSON, 'Roy. Soc. Phil. Trans.,' vol. 174, Part I., p. 132.

modulus in these wires is $-0{\cdot}00045$,* whilst for resistivity it is $+0{\cdot}0035$ approximately.

Density.

6. When a steel rod is drawn into wire the variation of density as the drawing proceeds may not progress uniformly, for it is not unlikely that the stress required to force the rod through the draw plate may so far separate the molecules longitudinally that the lateral compression does not compensate for the extension. In this case the density will diminish, and, in short, density will depend upon the ratio of extension to compression. It is, therefore, not only interesting, but of some importance to trace the change of density at each stage of manufacture and to compare this with the change of the properties already examined.

The method adopted was to weigh a suitable length of the wire in air and afterwards in distilled water, at the same time noting the data necessary for corrections on account of density of the water, the buoyancy of the air, the weight of the suspending fibre, etc., the most important correction being the one which makes allowance for the temperature of the water differing from that of its greatest density, approximately 4°.

For the sake of confirmation an entirely independent duplicate set of observations was carried out, and the agreement between the two sets was very satisfactory, especially in the earlier specimens, which, being of greater size and mass, could be weighed with relatively higher accuracy. The results are given in the last column of Table IX. It will be seen that they end at the eleventh wire, as the twelfth was accidentally mislaid; this is particularly unfortunate, as it would have been useful to know whether No. 12 exhibited a density greater or less than the abnormally high density of No. 11.

The feature which first claims notice is the diminution of density, although only slight, which takes place when the rod is first subjected to traction, namely, between the 2nd and 3rd, and between the 4th and 5th stages.

This appears to be an illustration of the remarks at the beginning of this section. Afterwards, as the drawing progresses, there is a steady increase of density, with a large increment between the 10th and 11th stages. The entire variation of density is about 2·5 per cent., the least and greatest values lying respectively at the beginning and end of the list.

As a general rule, it has been observed that density and Young's modulus in steel vary directly together, and this leads to a comparison of the present results with the curve of the modulus. The similarity is not immediately obvious, but, in both, annealing and tempering produce an upward movement, whilst the first drawing after either of these operations produces very little change, subsequent drawings, however,

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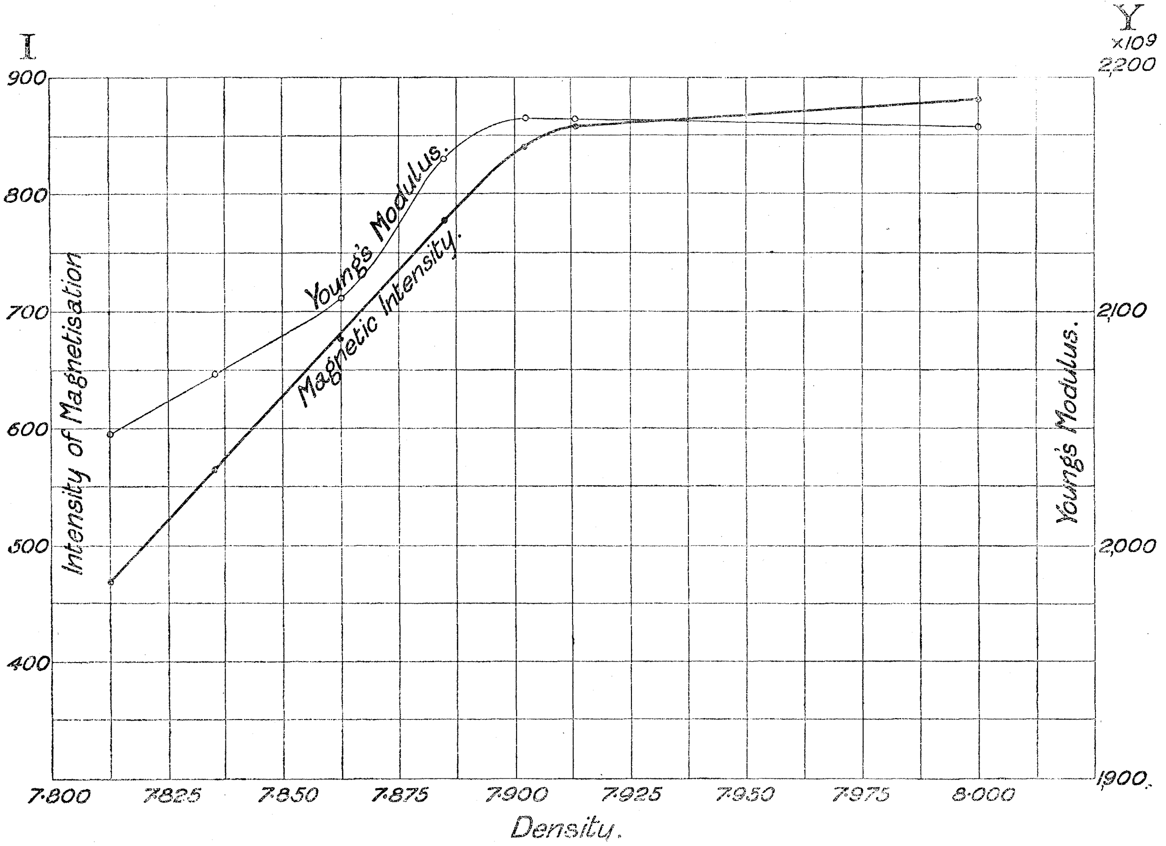
* According to Styffe the average value for ordinary steel is 0·0003. "Strength of Iron and Steel," by Knut Styffe, p. 122. *Vide* also Tomlinson, 'Roy. Soc. Phil. Trans.,' vol. 179, p. 23; vol. 174, pp. 132–133.

bending the curves upwards rapidly. But the final drop in the modulus does not appear to have a counterpart here, unless the missing No. 12 diminishes in density.

It is to be noticed that the percentage variation of YOUNG'S modulus from one end of the curve to the other is about ten times the percentage variation of the density, and also the average temperature coefficient of the modulus is about ten times the temperature coefficient of density, or cubical expansion, so that it is possible that much of the diminution of elasticity with rise of temperature may be due to thermal expansion diminishing the density.

A more obvious correspondence exists between the curve of residual magnetic intensity (dimension ratio = 100) and the curve of density, both rising continually and rapidly from the 5th point. The relationship is more easily traced when intensity is plotted against density, as in Diagram IX. Between the 5th and 9th stages inclusive the ratio of the increment of magnetic moment per unit volume to the increment of mass per unit volume is nearly constantly 4100, or each molecule added per unit volume contributes directly or indirectly to the whole a magnetic moment 4100 times its mass. This is, however, a doubtful clue to even an inferior limit to the magnetic moment of a molecule, since it cannot be assumed that there is

Diagram IX.



The relation of residual magnetic intensity and YOUNG'S modulus to density in drawn steel.

an invariable structure maintained throughout the progress of the drawing. Indeed, it is not difficult to see from the fracture of the wires that as the drawing proceeds a fibrous structure is developed for several stages after tempering, and the formation of this fibrous structure may be of importance in augmenting the magnetic intensity.

For the sake of comparison, YOUNG's modulus has been plotted on Diagram IX., the points being taken from a smooth curve of the modulus and traction, and the curve bears out the statement that an increase of density in general improves elastic properties. It also shows that elasticity and magnetic intensity are correlated.

To complete this part of the investigation of the properties of drawn steel, it was intended to add an account of the changes which might take place in the cubical expansion of these wires, and to trace the connection of these with other changes, but although some preliminary experiments have been made, the results are not yet sufficiently advanced to be presented.

This investigation has been carried out at the Owens College, Manchester, at intervals during the last three or four years, and I am greatly indebted to Professor SCHUSTER for allowing me to avail myself of the facilities for research which the Physical Laboratory there provides.

TABLE IX.—Influence of Drawing on Resistivity, YOUNG's Modulus, and their Temperature Coefficients, and Density.

No.	Condition.	Resistivity (at air temperature, about 16°). Ohms per centimetre cube, ρ.	Temperature coefficient of resistivity, α.	YOUNG's modulus (at air temperature), Y.	Temperature coefficient of YOUNG's modulus, γ.	Density. Grams. per cubic centimetre at 16°.
		$\times 10^{-5}$	$\times 10^{-3}$	$\times 10^{11}$	$\times 10^{-4}$	
1	Rolled Rod	1·601	+3·43	1·78	− 3·35	7·803
2	Annealed	1·796	+2·94	1·83	− 1·64	7·827
3	Hard drawn	1·585	+4·66	1·85	−10·2	7·815
4	Tempered	1·784	+3·49	2·04	− 2·30	7·818
5	Cold drawn	1·716	+3·55	2·04	− 3·79	7·813
6	,,	1·657	+3·48	2·08	− 5·90	7·835
7	,,	1·645	+3·61	2·11	− 4·87	7·871
8	,,	1·638	+3·51	2·20	− 2·79	7·881
9	,,	1·627	+3·43	2·13	− 6·53	7·902
10	,,	1·633	+3·52	2·21	− 1·79	7·913
11	,,	1·696	+3·44	2·18	− 6·42	8·001
12	,,	1·738	+3·48	1·95	− 4·24	—
12*a*	,,	1·739	—	—	—	—
13*a*	,,	1·772	—	—	—	—
14*a*	,,	1·876	—	—	—	—
	Glass hard	2·760	+1·77	—	—	7·740